Computer Science 121

Problem Set 1

due 6 October 2007

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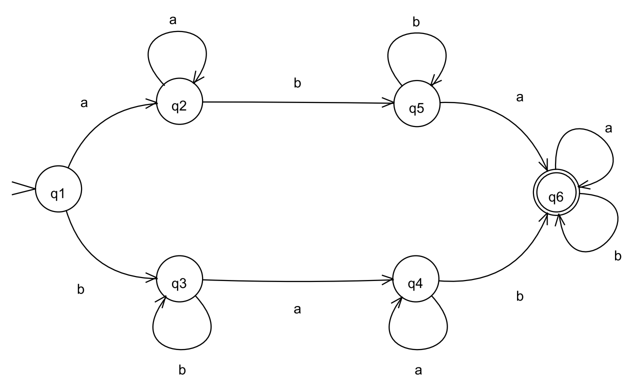
No collaborators.

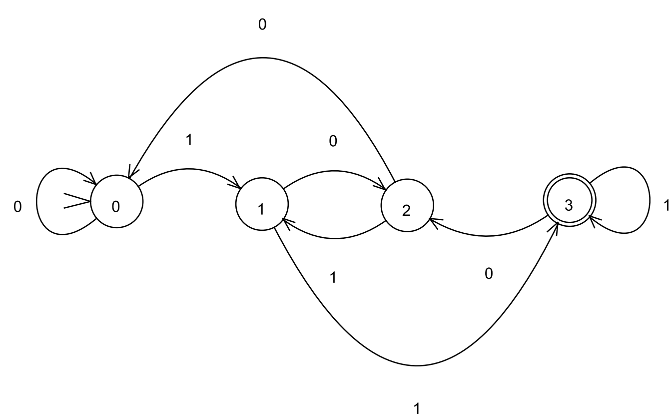
1. (A) This automata accepts only $(ab)^n a$, $n \in \mathbb{N}$ where power is defined by repeated concatnations.

   (B) All non-empty strings which do not contain substrings aa, bb.

   (C) The following recognizes strings with both $\{ab, ba\}$



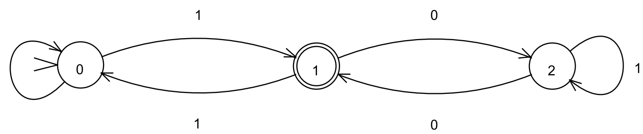
2. (A) The following recognizes binary strings which fulfil the condition $x \bmod 4 = 3$:



(B) $N = (\{0,1,2,3\},\{0,1\},\delta,0,\{3\})$, where δ is defined by

	0	1	ϵ
0	$\{0\}$	$\{1\}$	$\{0\}$
1	$\{2\}$	$\{3\}$	$\{1\}$
2	$\{0\}$	$\{1\}$	$\{2\}$
3	$\{2\}$	$\{3\}$	$\{3\}$

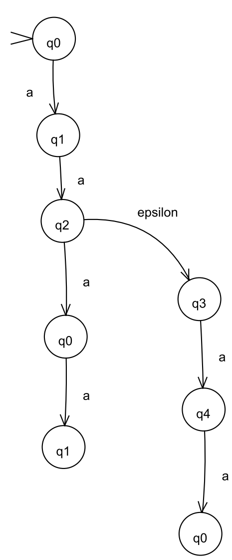
(C) The following accepts binary numerals leaving a remainder 1 when divided by 3:



(D) Assuming Big-ending encoding (tape is read most significant bit/base first), we can recursively construct any number by appending a digit onto the least significant end of the number. This implies that for each new digit d added to number n the resulting remainder (under modulus m and base k) can be expressed as $nk + d \bmod m$. This is guranteed to be in the set $\{0, 1...m - 1\}$ and thus we can construct a DFA of size m (order of this set). This is also the minimum size that can be constructed, as if there is any DFA of size less than m then it will be possible to append a digit such that the DFA will not have an appropriate state.

3. For the base case, we have $n = 1 = F_1$. Now for some $n \in \mathbb{N}$ consider F_n defined as the largest Fibonacci less than n. Assume that for $n = F_n + k$, k can be composed of a sum of Fibonacci numbers. By definition of fibonacci numbers we can say $F_{n+1} = F_n + F_{n-1}$ By definition, $F_{n+1} > n$ and therefore $F_n + F_{n-1} > n$ and $k = n - F_n$ which implies $k < F_{n-1}$ Which shows that the inductive hypothesis holds.

4. (A) The tree for this diagram is given by



(B) String $aaaa$ is not accepted since $aaaa$ ends with state q_1 and $aa\epsilon aa$ ends with state q_0. String $aaaaa$ on the other hand is accepted (assuming no ϵ branching).

(C) $aa\{aaaa\}^*$ or $aa\{aaa\}^*$

5. (A) Given a DFA, the number of states is a finite set given by $T = \{q_1, q_2, ..., q_n\}$ then we can trace the moves with $S = \{q_{i_1}, q_{i_2}, ...\}$ By the pigeonhole principal we can say that the maximum order of the set S is $|T|$ if no element is repeated (ie there exist no cycles) Since $|T|$ is finite (by definition), $|S|$ is finite.

   (B) This statement is false, consider the languages $L_1 = \{a, b\}, L_2 = \{ab, ba, aa, bb\}$ then $(L_1 \cap L_2)^* = \{\epsilon\}$ but $L_1^* \cap L_2^* = L_1^* = L_2^*$

6. First, Consider set R_n as the set of characters of string length n with odd number of as. We can construct strings of length $n+1$ via appending a character to the end. Any strings in R_n can become a valid string of length $n+1$ by appending an a to the end. Any strings in S_n becomes a valid string in S_{n+1} by appending a b or c. This implies that $S_{n+1} = R_n + 2S_n$, Also, $R_n + S_n$ is all combinations of 3 character strings, which is $R_n + S_n = 3^n$, by substition we have $S_{n+1} = 3^n + S_n$ which is the same as the formulation before. Next, this can be turned into $S_n = \frac{3^n+1}{2}$ by simple intuition (which is confirmed by the online encylcopedia of integer sequences)

7. Assume otherwise, Consider a group of n nodes (people) with bidirectional arcs (knowing a person). In this case the maximum degree per node will be $n-1$, however, there are n nodes to assign degrees. This means that one of the degree values must overlap by the pigeonhole principle, which is equivalent to two people knowing the same number of people.